

regression_lineaire

October 16, 2021

1 Régression linéaire

Ce notebook s'intéresse à la façon d'interpréter les résultats d'une régression linéaire lorsque les variables sont corrélées puis il explore une façon d'associer arbre de décision et régression linéaire pour construire une régression linéaire par morceaux.

```
[1]: from jupyterhelper import add_notebook_menu
add_notebook_menu()
```

```
[1]: <IPython.core.display.HTML object>
```

```
[2]: %matplotlib inline
```

1.1 Un cas simple

Une façon d'interpréter des résultats statistiques est de les calculer dans un cas où la réponse cherchée est connue. On simule un modèle simple $Y = \alpha X_1 + 0.X_2 + \epsilon$ et on cale une régression linéaire. On suppose que X_1, X_2, ϵ sont des variables aléatoires gaussiennes de même variance et moyenne.

```
[3]: import numpy.random as npr
eps = npr.normal(1000)
X = npr.normal(size=(1000, 3))
alpha = 2
Y = alpha * X[:,0] + X[:, 2]
X.shape, Y.shape
```

```
[3]: ((1000, 3), (1000,))
```

```
[4]: from numpy import corrcoef
corrcoef(X.T)
```

```
[4]: array([[ 1.          , -0.0312982 ,  0.05188551],
          [-0.0312982 ,  1.          , -0.00356494],
          [ 0.05188551, -0.00356494,  1.          ]])
```

```
[5]: from statsmodels.regression.linear_model import OLS
```

```
[6]: model = OLS(Y,X[:, :2])
results = model.fit()
su = results.summary()
su
```

```
[6]: <class 'statsmodels.iolib.summary.Summary'>
      """
                OLS Regression Results
      =====
Dep. Variable:          y      R-squared:                0.815
Model:                  OLS    Adj. R-squared:           0.815
Method:                 Least Squares  F-statistic:            2204.
Date:                   Mon, 15 Oct 2018  Prob (F-statistic):       0.00
Time:                   10:34:12    Log-Likelihood:         -1385.2
No. Observations:      1000    AIC:                    2774.
Df Residuals:          998      BIC:                    2784.
Df Model:               2
Covariance Type:       nonrobust
      =====
                coef      std err          t      P>|t|      [0.025      0.975]
-----
x1                2.0519      0.031     66.347      0.000       1.991      2.113
x2               -0.0032      0.033     -0.097      0.922      -0.067      0.061
      =====
Omnibus:              0.709    Durbin-Watson:          1.990
Prob(Omnibus):        0.701    Jarque-Bera (JB):       0.674
Skew:                 0.063    Prob(JB):               0.714
Kurtosis:             3.010    Cond. No.                1.07
      =====

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
      """
```

```
[7]: results.rsquared, results.rsquared_adj
```

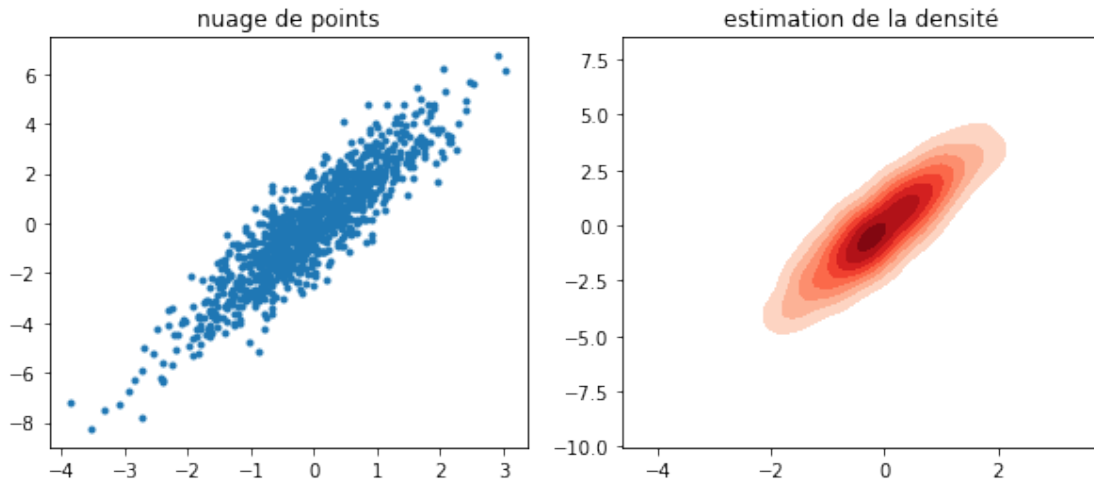
```
[7]: (0.8153831029946165, 0.8150131292531227)
```

On vérifie que le coefficient devant X_1 est non nul (P-value nulle, 0 n'est pas l'intervalle de confiance). Le coefficient devant X_2 n'est pas nul mais presque, la P-value est élevée, le coefficient R^2 est élevé. Dessinons.

```
[8]: import matplotlib.pyplot as plt
import seaborn
fig, ax = plt.subplots(1, 2, figsize=(10,4))
ax[0].plot(X[:, 0], Y, '.')
seaborn.kdeplot(X[:, 0], Y, cmap="Reds", shade=True, shade_lowest=False, ax=ax[1])
ax[0].set_title("nuage de points")
ax[1].set_title("estimation de la densité");
```

```
c:\python370_x64\lib\site-packages\scipy\stats\stats.py:1713: FutureWarning:
Using a non-tuple sequence for multidimensional indexing is deprecated; use
`arr[tuple(seq)]` instead of `arr[seq]`. In the future this will be interpreted
as an array index, `arr[np.array(seq)]`, which will result either in an error or
a different result.
```

```
    return np.add.reduce(sorted[indexer] * weights, axis=axis) / sumval
```

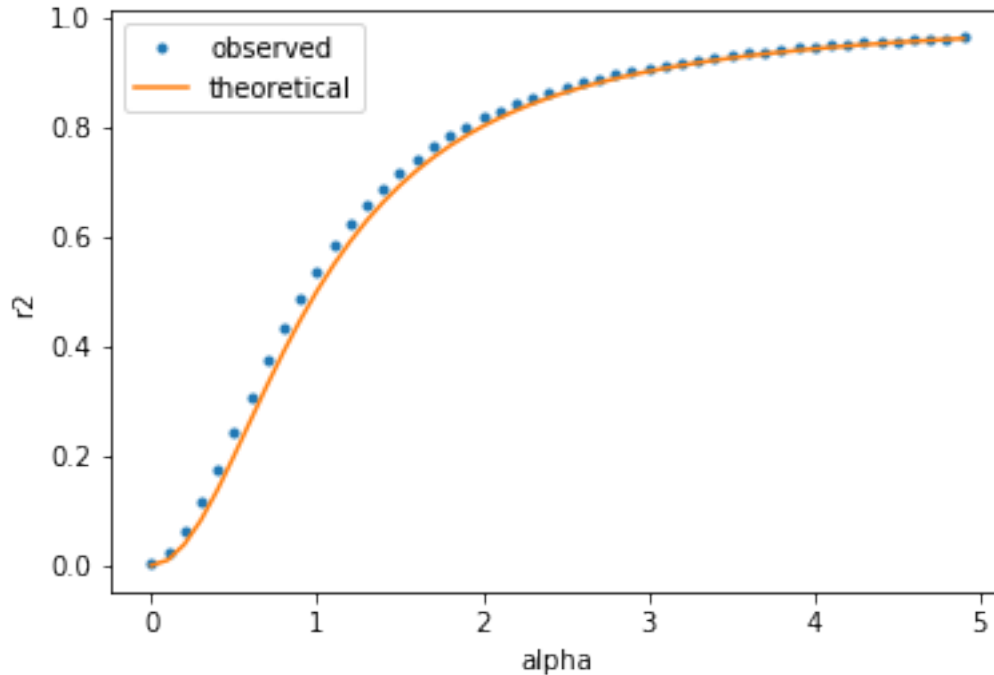


1.2 Evolution de R²

Dans la régression précédente, le coefficient R^2 transcrit en quelque sorte la part du bruit ϵ par rapport au terme αX_1 . Faisons varier α .

```
[9]: alphas = []
      r2s = []
      for a in [0.1 * i for i in range(0, 50)]:
          Y = a*X[:,0] + X[:, 2]
          model = OLS(Y,X[:, :2])
          results = model.fit()
          alphas.append(a)
          r2s.append(results.rsquared)
```

```
[10]: fig, ax = plt.subplots(1, 1)
       ax.plot(alphas, r2s, '.', label="observed")
       ax.plot(alphas, [a**2/(1+a**2) for a in alphas], label='theoretical')
       ax.set_xlabel("alpha")
       ax.set_ylabel("r2")
       ax.legend();
```



Dans ce cas de régression simple, la valeur à prédire est y_i , la valeur prédite est $\hat{y}_i = \alpha X_{1i}$ et la moyenne $\bar{y} = \alpha \bar{X}_1 + \bar{\epsilon} = 0$.

$$R^2 = 1 - \frac{\sum_{i=1}^n (\hat{y}_i - \bar{y})^2}{\sum_{i=1}^n (y_i - \bar{y})^2} = 1 - \frac{\mathbb{V}\epsilon}{\alpha^2 \mathbb{V}X_1 + \mathbb{V}\epsilon} = 1 - \frac{1}{1 + \alpha^2} = \frac{\alpha^2}{1 + \alpha^2}$$

1.3 Deux variables corrélées

On ne change pas le modèle mais on fait en sorte que $X_2 = X_1$. Les deux variables sont corrélées.

```
[11]: X[:, 1] = X[:, 0]
      Y = 2*X[:,0] + X[:, 2]
      model = OLS(Y,X[:, :2])
      results = model.fit()
      results.summary()
```

```
[11]: <class 'statsmodels.iolib.summary.Summary'>
      """
```

```

                                OLS Regression Results
=====
Dep. Variable:                    y      R-squared:                0.810
Model:                            OLS      Adj. R-squared:           0.810
Method:                            Least Squares      F-statistic:             4271.
Date:                            Mon, 27 Nov 2017      Prob (F-statistic):      0.00
Time:                            12:06:03      Log-Likelihood:         -1411.2
No. Observations:                  1000      AIC:                    2824.
Df Residuals:                      999      BIC:                    2829.
Df Model:                          1
Covariance Type:                   nonrobust
=====
```

	coef	std err	t	P> t	[0.025	0.975]
x1	1.0288	0.016	65.349	0.000	0.998	1.060
x2	1.0288	0.016	65.349	0.000	0.998	1.060

Omnibus:	8.165	Durbin-Watson:	1.944
Prob(Omnibus):	0.017	Jarque-Bera (JB):	6.024
Skew:	-0.064	Prob(JB):	0.0492
Kurtosis:	2.642	Cond. No.	1.61e+16

Warnings:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

[2] The smallest eigenvalue is 7.69e-30. This might indicate that there are strong multicollinearity problems or that the design matrix is singular.

""

```
[12]: model.rank
```

```
[12]: 1
```

Les variables corrélées n'ont pas l'air de déranger l'algorithme de résolution car il utilise la méthode [SVD](#) pour résoudre le même problème dans un espace de moindre dimension. Le problème survient que les deux variables ne sont pas complètement corrélées. On étudie le modèle $Y \sim X_1 + X_2'$ avec $X_2' = \alpha X_1 + (1 - \alpha)X_2$ et on réduit la variance du bruit pour en diminuer les effets.

```
[13]: X_ = npr.normal(size=(1000, 3))
```

```
[14]: alphas = [0.9 + i * 0.01 for i in range(0,11)]
res = []
for a in alphas:
    X = X_.copy()
    X[:, 1] = a * X[:, 0] + (1-a) * X[:, 1]
    Y = X[:, 0] + X[:, 1] + 0.1 * X[:, 2]
    model = OLS(Y,X[:, :2])
    results = model.fit()
    res.append(dict(alpha=a, r2=results.rsquared, rank=model.rank, c1=results.
↳params[0], c2=results.params[1]))

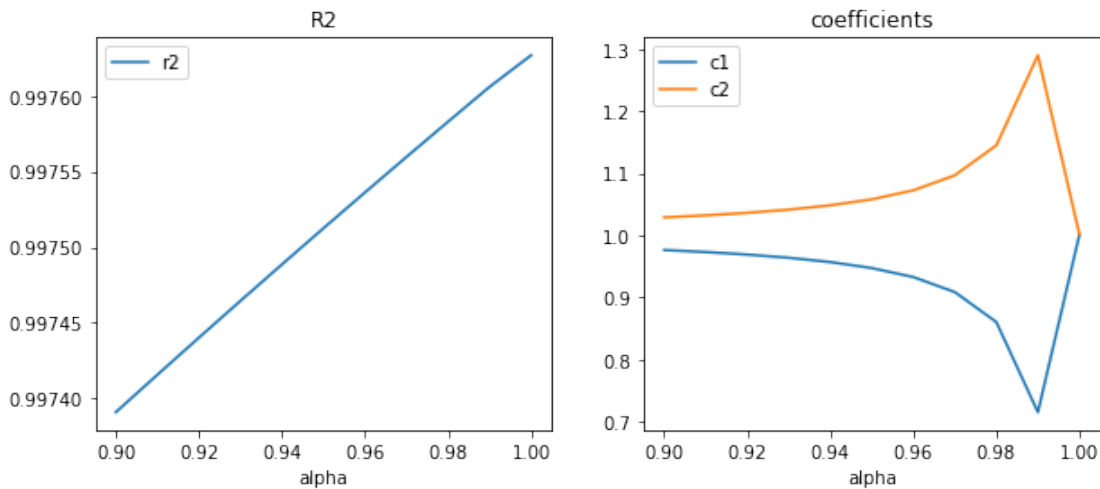
import pandas
df = pandas.DataFrame(res)
df = df.set_index('alpha')
df
```

```
[14]:
```

alpha	c1	c2	r2	rank
0.90	0.976370	1.028982	0.997391	2
0.91	0.973150	1.032202	0.997416	2
0.92	0.969125	1.036227	0.997440	2
0.93	0.963950	1.041402	0.997464	2
0.94	0.957049	1.048303	0.997489	2
0.95	0.947389	1.057963	0.997513	2
0.96	0.932898	1.072454	0.997536	2

0.97	0.908747	1.096605	0.997560	2
0.98	0.860444	1.144908	0.997583	2
0.99	0.715536	1.289816	0.997606	2
1.00	1.001279	1.001279	0.997627	1

```
[15]: fig, ax = plt.subplots(1,2, figsize=(10,4))
df[["r2"]].plot(ax=ax[0])
df[["c1", "c2"]].plot(ax=ax[1])
ax[0].set_title("R2")
ax[1].set_title("coefficients");
```



Le r^2 augmente quand la corrélation augmente mais les coefficients sont moins fiables. Les résultats devraient être sensiblement identiques en théorie mais en pratique, plus le déterminant devient proche de zéro, plus l'ordinateur est limité par sa précision numérique. Pour en savoir plus, vous pouvez lire un examen écrit que j'ai rédigé, en python bien sûr : [Examen Programmation ENSAE première année 2006](#). Cette précision est aux alentours de 10^{-15} ce qui correspond à la précision numérique des `double`.

```
[16]: alphas = [1 - 10**(-i) for i in range(10,18)]
res = []
for a in alphas:
    X = X_.copy()
    X[:, 1] = a * X[:, 0] + (1-a) * X[:, 1]
    Y = X[:, 0] + X[:, 1] + X[:, 2]
    model = OLS(Y,X[:, :2])
    results = model.fit()
    res.append(dict(alpha_1=a-1, r2=results.rsquared, rank=model.rank, c1=results.
    ↪params[0], c2=results.params[1]))

import pandas
df = pandas.DataFrame(res)
df = df.set_index('alpha_1')
df
```

```
[16]:          c1          c2          r2  rank
alpha_1
```

```

-1.000000e-10 -2.898180e+08  2.898180e+08  0.811519    2
-1.000000e-11 -2.898201e+09  2.898201e+09  0.811519    2
-9.999779e-13 -2.898941e+10  2.898941e+10  0.811519    2
-1.000311e-13 -2.891422e+11  2.891422e+11  0.811518    2
-9.992007e-15 -2.915101e+12  2.915101e+12  0.811508    2
-9.992007e-16  1.012789e+00  1.012789e+00  0.811359    2
-1.110223e-16  1.012789e+00  1.012789e+00  0.811359    1
 0.000000e+00  1.012789e+00  1.012789e+00  0.811359    1

```

On fait un dernier test avec `scikit-learn` pour vérifier que l'algorithme de résolution donne des résultats similaires pour un cas où le déterminant est quasi-nul.

```

[17]: from sklearn.linear_model import LinearRegression
      from sklearn.metrics import r2_score

alphas = [0.9 + i * 0.01 for i in range(0,11)]
res = []
for a in alphas:
    X = X_.copy()
    X[:, 1] = a * X[:, 0] + (1-a) * X[:, 1]
    Y = X[:, 0] + X[:, 1] + X[:, 2]
    model = LinearRegression()
    model.fit(X[:, :2], Y)
    r2 = r2_score(Y, model.predict(X[:, :2]))
    res.append(dict(alpha=a, c1=model.coef_[0], c2=model.coef_[1], r2=r2))

import pandas
df = pandas.DataFrame(res)
df = df.set_index('alpha')
df

```

```

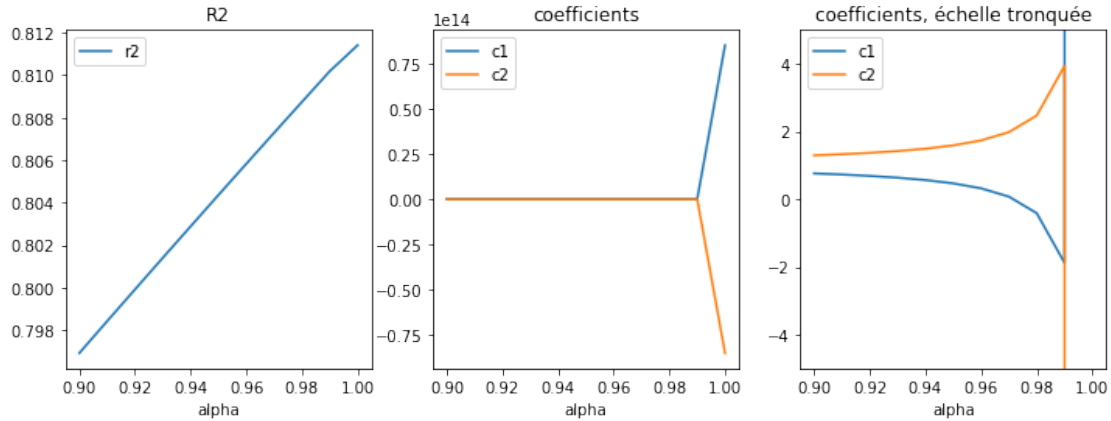
[17]:
      alpha          c1          c2          r2
0.90  7.601931e-01  1.293903e+00  0.796916
0.91  7.275372e-01  1.326559e+00  0.798417
0.92  6.867173e-01  1.367379e+00  0.799911
0.93  6.342346e-01  1.419862e+00  0.801399
0.94  5.642576e-01  1.489839e+00  0.802880
0.95  4.662898e-01  1.587807e+00  0.804355
0.96  3.193382e-01  1.734758e+00  0.805823
0.97  7.441878e-02  1.979678e+00  0.807283
0.98 -4.154200e-01  2.469516e+00  0.808736
0.99 -1.884936e+00  3.939033e+00  0.810182
1.00  8.512221e+13 -8.512221e+13  0.811404

```

```

[18]: fig, ax = plt.subplots(1,3, figsize=(12,4))
      df[["c1", "c2"]].plot(ax=ax[1])
      df[["c1", "c2"]].plot(ax=ax[2])
      df[["r2"]].plot(ax=ax[0])
      ax[0].set_title("R2")
      ax[1].set_title("coefficients")
      ax[2].set_ylim([-5, 5])
      ax[2].set_title("coefficients, échelle tronquée");

```



Le second graphe est trompeur mais il ne faut pas oublier de regarder l'échelle de l'axe des ordonnées.

1.4 Indicatrices

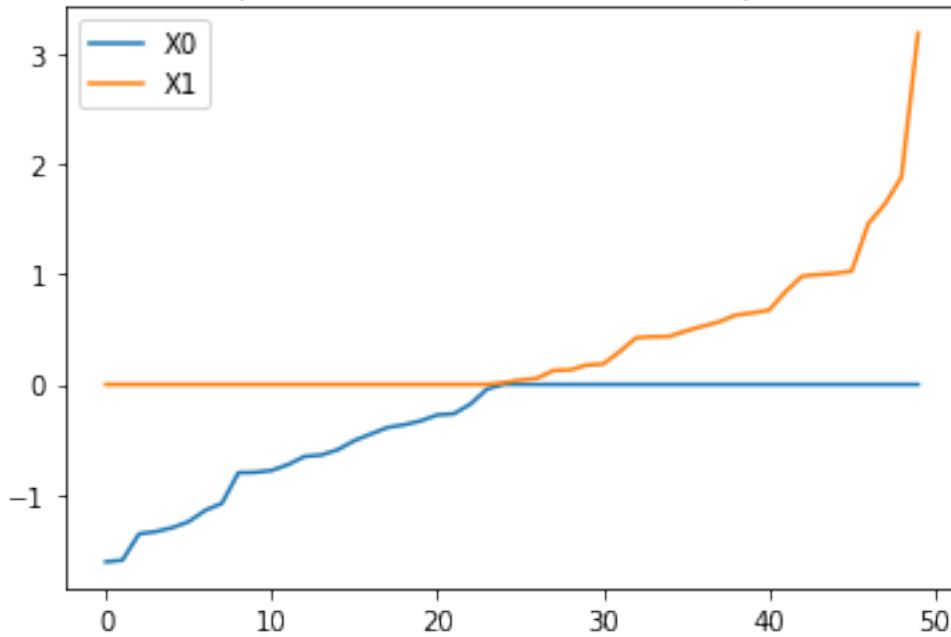
X_1 est une variable aléatoire gaussienne. On teste maintenant un modèle $Y = X'_1 + X'_2 + \epsilon$ avec $X'_1 = X_1 \mathbf{1}_{X_1 < 0}$ et $X'_2 = X_1 \mathbf{1}_{X_1 \geq 0}$.

```
[19]: X = npr.normal(size=(1000, 3))
X[:, 1] = X[:, 0]
X[X[:, 0] >= 0, 0] = 0
X[X[:, 1] < 0, 1] = 0
Y = X[:, 0] + X[:, 1] + X[:, 2]
corrcoef(X.T)
```

```
[19]: array([[ 1.          ,  0.47358312, -0.03083914],
          [ 0.47358312,  1.          , -0.01293737],
          [-0.03083914, -0.01293737,  1.          ]])
```

```
[20]: from pandas import DataFrame
names = ["X%d" % i for i in range(X.shape[1]-1)]
ax = DataFrame(X[:50,:2], columns=names).sort_values(names).reset_index(drop=True).
    ↪plot()
ax.set_title("Représentation des features tronquées");
```


Représentation des features tronquées



```
[21]: model = OLS(Y,X[:, :3])
      results = model.fit()
      results.summary()
```

```
[21]: <class 'statsmodels.iolib.summary.Summary'>
      """
```

```

=====
                        OLS Regression Results
=====
Dep. Variable:          y      R-squared:                1.000
Model:                  OLS    Adj. R-squared:           1.000
Method:                 Least Squares  F-statistic:             2.212e+33
Date:                   Mon, 15 Oct 2018  Prob (F-statistic):       0.00
Time:                   10:56:29      Log-Likelihood:          33713.
No. Observations:      1000          AIC:                    -6.742e+04
Df Residuals:          997           BIC:                    -6.740e+04
Df Model:               3
Covariance Type:       nonrobust
=====

```

	coef	std err	t	P> t	[0.025	0.975]
x1	1.0000	2.42e-17	4.14e+16	0.000	1.000	1.000
x2	1.0000	2.39e-17	4.18e+16	0.000	1.000	1.000
x3	1.0000	1.73e-17	5.78e+16	0.000	1.000	1.000

```

=====
Omnibus:                4.249    Durbin-Watson:           2.031
Prob(Omnibus):          0.119    Jarque-Bera (JB):        4.338
Skew:                   -0.107   Prob(JB):                 0.114
Kurtosis:                3.242    Cond. No.:                1.40
=====

```

Warnings:

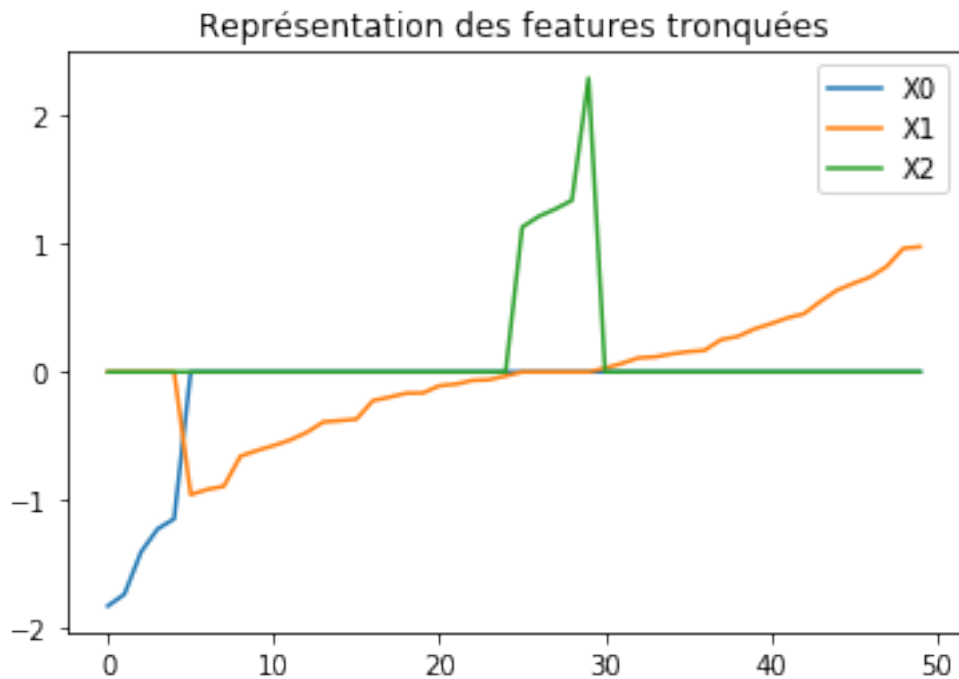
```
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
"""
```

On découpe en trois.

```
[22]: import numpy
X = npr.normal(size=(1000, 4))
for i in range(0, 3):
    X[:, i] = X[:, 0]
X[:, 3] = X[:, 2]
X[X[:, 0] > -1, 0] = 0
X[(X[:, 0] < -1) | (X[:, 0] > 1), 1] = 0
X[X[:, 0] < 1, 2] = 0
Y = X[:, 0] + X[:, 1] + X[:, 2] + X[:, 3]
corrcoef(X.T)
```

```
[22]: array([[ 1.          , -0.00347584,  0.16846101,  0.06722762],
        [-0.00347584,  1.          ,  0.00326437, -0.04707208],
        [ 0.16846101,  0.00326437,  1.          ,  0.08754832],
        [ 0.06722762, -0.04707208,  0.08754832,  1.          ]])
```

```
[23]: from pandas import DataFrame
names = ["X%d" % i for i in range(X.shape[1]-1)]
ax = DataFrame(X[:50,:3], columns=names).sort_values(names).reset_index(drop=True).
    ↪plot()
ax.set_title("Représentation des features tronquées");
```



```
[24]: model = OLS(Y,X[:, :4])
      results = model.fit()
      results.summary()
```

```
[24]: <class 'statsmodels.iolib.summary.Summary'>
      """
```

```

                                OLS Regression Results
=====
Dep. Variable:                    y      R-squared:                    1.000
Model:                            OLS      Adj. R-squared:                1.000
Method:                            Least Squares      F-statistic:                    1.910e+32
Date:                            Mon, 15 Oct 2018      Prob (F-statistic):              0.00
Time:                            10:57:27      Log-Likelihood:                  32608.
No. Observations:                  1000      AIC:                            -6.521e+04
Df Residuals:                      996      BIC:                            -6.519e+04
Df Model:                          4
Covariance Type:                  nonrobust
=====

```

	coef	std err	t	P> t	[0.025	0.975]
x1	1.0000	8.75e-17	1.14e+16	0.000	1.000	1.000
x2	1.0000	1.22e-16	8.23e+15	0.000	1.000	1.000
x3	1.0000	8.33e-17	1.2e+16	0.000	1.000	1.000
x4	1.0000	5.23e-17	1.91e+16	0.000	1.000	1.000

```

=====
Omnibus:                          457.967      Durbin-Watson:                  1.816
Prob(Omnibus):                     0.000      Jarque-Bera (JB):                1967.636
Skew:                              -2.198      Prob(JB):                        0.00
Kurtosis:                          8.282      Cond. No.                        2.35
=====

```

```

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
      """
```

1.5 Régression linéaire par morceaux

On se place dans un cas particulier où le problème est linéaire par morceaux :

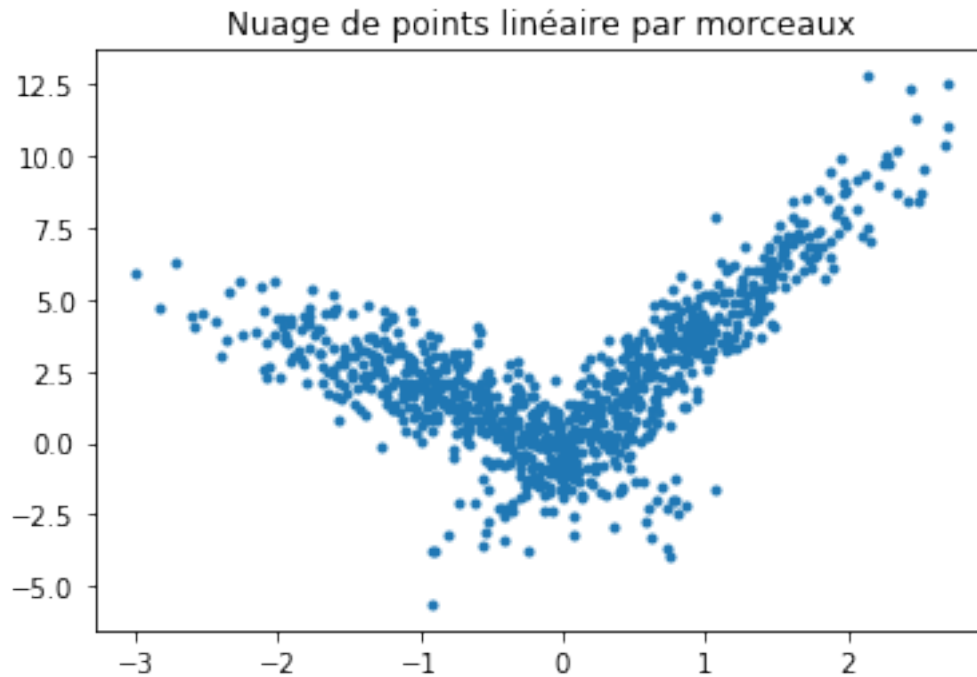
$$Y = -2X_1 \mathbf{1}_{X_1 + \epsilon_1 < 0} + 4X_1 \mathbf{1}_{X_1 + \epsilon_1 > 0} + \epsilon_2$$

La régression donne de très mauvais résultat sur ce type de problèmes mais on cherche une façon systématique de découper le problème en segments linéaires.

```
[25]: X = npr.normal(size=(1000,4))
      alpha = [4, -2]
      t = (X[:, 0] + X[:, 3] * 0.5) > 0
      switch = numpy.zeros(X.shape[0])
      switch[t] = 1
      Y = alpha[0] * X[:, 0] * t + alpha[1] * X[:, 0] * (1-t) + X[:, 2]
```

```
[26]: fig, ax = plt.subplots(1, 1)
      ax.plot(X[:, 0], Y, ".")
```

```
ax.set_title("Nuage de points linéaire par morceaux");
```



```
[27]: model = OLS(Y,X[:, :1])
      results = model.fit()
      results.summary()
```

```
[27]: <class 'statsmodels.iolib.summary.Summary'>
      """
```

```

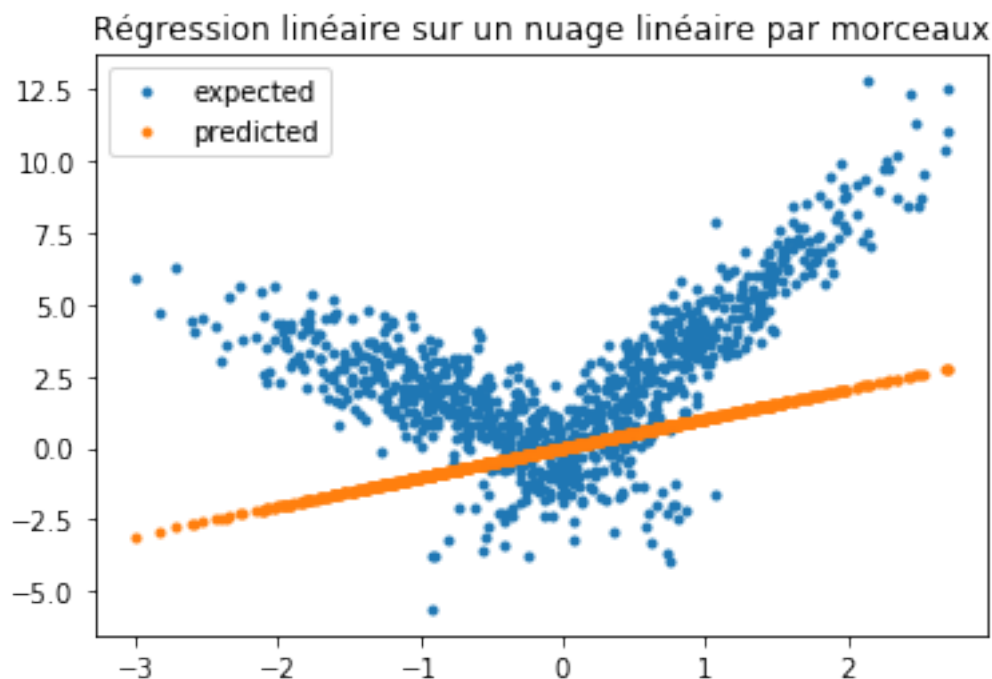
                                OLS Regression Results
=====
Dep. Variable:                  y      R-squared:                0.094
Model:                          OLS      Adj. R-squared:           0.093
Method:                         Least Squares      F-statistic:              104.0
Date:                            Mon, 15 Oct 2018      Prob (F-statistic):       2.69e-23
Time:                             10:59:28      Log-Likelihood:           -2594.9
No. Observations:                1000      AIC:                      5192.
Df Residuals:                     999      BIC:                      5197.
Df Model:                          1
Covariance Type:                  nonrobust
=====
              coef      std err          t      P>|t|      [0.025      0.975]
-----
x1              1.0252      0.101      10.197      0.000      0.828      1.222
=====
Omnibus:                        3.882      Durbin-Watson:            1.092
Prob(Omnibus):                   0.144      Jarque-Bera (JB):         3.834
Skew:                             0.151      Prob(JB):                  0.147
Kurtosis:                        3.015      Cond. No.                  1.00

```

```
=====
Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
"""
```

```
[28]: yp = results.predict(X[:, :1])
```

```
[29]: fig, ax = plt.subplots(1, 1)
ax.plot(X[:, 0], Y, ".", label="expected")
ax.plot(X[:, 0], yp, ".", label="predicted")
ax.legend()
ax.set_title("Régression linéaire sur un nuage linéaire par morceaux");
```

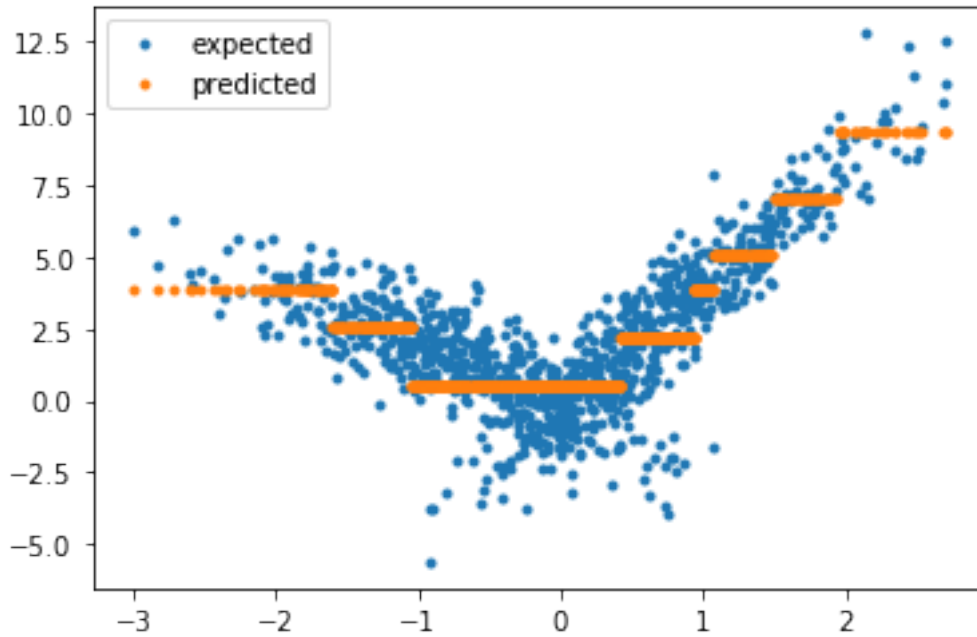


Passons à un arbre de décision qui n'est pas le meilleur modèle mais on va détourner ses résultats pour revenir à un problème de régression par morceaux.

```
[30]: from sklearn.tree import DecisionTreeRegressor
model = DecisionTreeRegressor(min_samples_leaf=10, max_depth=3)
model.fit(X[:, :1], Y)
yp = model.predict(X[:, :1])
```

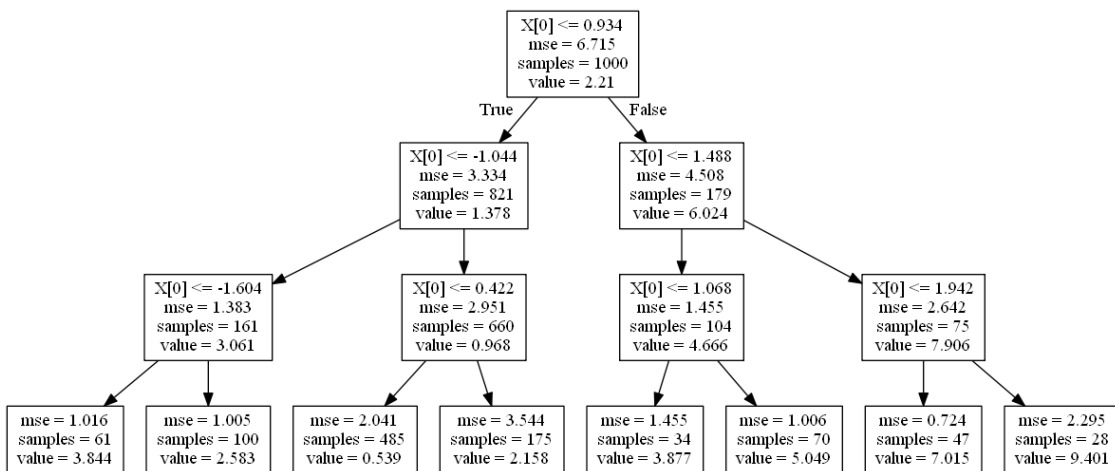
```
[31]: fig, ax = plt.subplots(1, 1)
ax.plot(X[:, 0], Y, ".", label="expected")
ax.plot(X[:, 0], yp, ".", label="predicted")
ax.legend()
r2 = r2_score(Y, model.predict(X[:, :1]))
ax.set_title("Arbre de décision sur un nuage linéaire par morceaux\nR2=%f" % r2);
```

Arbre de décision sur un nuage linéaire par morceaux R2=0.703533



```
[32]: from sklearn.tree import export_graphviz
export_graphviz(model, out_file="arbre.dot")
from pyensae.graphhelper import run_dot
run_dot("arbre.dot", "arbre.png")
from IPython.display import Image
Image("arbre.png")
```

[32]:



On extrait tous les seuils de l'arbre et on ajoute les milieux de segments.

```
[33]: th = list(sorted(set(model.tree_.threshold)))
th += [(th[i] + th[i-1])/2 for i in range(1,len(th))]
th = list(sorted(th))
th
```

```
[33]: [-2.0,
-1.8018612563610077,
-1.6037225127220154,
-1.323736995458603,
-1.0437514781951904,
-0.3109976723790169,
0.4217561334371567,
0.678125374019146,
0.9344946146011353,
1.0011553764343262,
1.067816138267517,
1.2776717841625214,
1.4875274300575256,
1.7147845923900604,
1.9420417547225952]
```

On fait une régression sur les variables $W_{i>0} = X_1 \mathbf{1}_{X_1 > t_i}$, $W_0 = X_1$ où les (t_i) sont les seuils.

```
[34]: W = numpy.zeros((X.shape[0], len(th)+1))
x = X[:, 0]
W[:, 0] = x
for i in range(len(th)):
    W[x > th[i], i+1] = x[x > th[i]]
```

```
[35]: model = OLS(Y,W)
results = model.fit()
results.summary()
```

```
[35]: <class 'statsmodels.iolib.summary.Summary'>
"""
```

```

                                OLS Regression Results
=====
Dep. Variable:                  y      R-squared:                0.849
Model:                            OLS      Adj. R-squared:           0.847
Method:                            Least Squares      F-statistic:               346.9
Date:                            Mon, 15 Oct 2018      Prob (F-statistic):        0.00
Time:                            11:07:03      Log-Likelihood:           -1697.7
No. Observations:                  1000      AIC:                      3427.
Df Residuals:                      984      BIC:                      3506.
Df Model:                          16
Covariance Type:                  nonrobust
=====

```

	coef	std err	t	P> t	[0.025	0.975]
x1	-1.8316	0.124	-14.806	0.000	-2.074	-1.589
x2	-0.0573	0.210	-0.273	0.785	-0.469	0.354
x3	-0.2440	0.235	-1.038	0.300	-0.705	0.217
x4	0.3291	0.218	1.508	0.132	-0.099	0.757
x5	-0.3610	0.206	-1.756	0.079	-0.764	0.042

x6	0.5528	0.197	2.811	0.005	0.167	0.939
x7	3.2628	0.395	8.253	0.000	2.487	4.039
x8	1.4566	0.449	3.244	0.001	0.576	2.338
x9	0.2701	0.311	0.869	0.385	-0.340	0.880
x10	0.7213	0.374	1.928	0.054	-0.013	1.456
x11	-0.4599	0.457	-1.006	0.315	-1.357	0.437
x12	0.4177	0.378	1.105	0.269	-0.324	1.159
x13	-0.2703	0.253	-1.069	0.285	-0.766	0.226
x14	0.5325	0.226	2.360	0.018	0.090	0.975
x15	-0.3703	0.229	-1.618	0.106	-0.819	0.079
x16	0.1996	0.194	1.026	0.305	-0.182	0.581

```
=====
Omnibus:                228.022   Durbin-Watson:           1.978
Prob(Omnibus):          0.000   Jarque-Bera (JB):       716.200
Skew:                   -1.110   Prob(JB):               3.01e-156
Kurtosis:               6.502   Cond. No.               36.3
=====
```

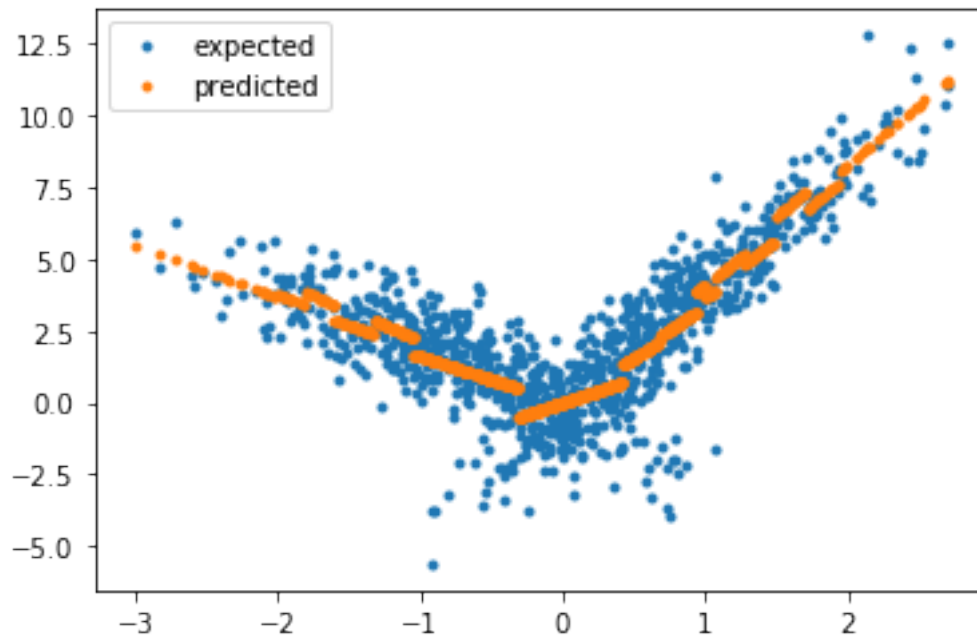
Warnings:

```
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
"""
```

Dessinons les résultats de la prédictions.

```
[36]: yp = results.predict(W)
fig, ax = plt.subplots(1, 1)
ax.plot(X[:, 0], Y, ".", label="expected")
ax.plot(X[:, 0], yp, ".", label="predicted")
ax.legend()
ax.set_title("Régression linéaire par morceaux\nsur un nuage linéaire par
↳morceaux\nR2=%f" % results.rsquared);
```


Régression linéaire par morceaux sur un nuage linéaire par morceaux R2=0.849426



Le modèle nous suggère de ne garder que quelques seuils. En s'appuyant sur les p-values :

```
[37]: keep = numpy.arange(len(results.pvalues))[results.pvalues < 0.05]
      keep
```

```
[37]: array([ 0,  5,  6,  7, 13])
```

```
[38]: W2 = W[:, keep]
```

```
[39]: model = OLS(Y,W2)
      results = model.fit()
      results.summary()
```

```
[39]: <class 'statsmodels.iolib.summary.Summary'>
      """
```

```
OLS Regression Results
=====
Dep. Variable:          y      R-squared:                0.846
Model:                  OLS    Adj. R-squared:           0.845
Method:                 Least Squares  F-statistic:             1094.
Date:                   Mon, 15 Oct 2018  Prob (F-statistic):       0.00
Time:                   11:07:38    Log-Likelihood:          -1708.6
No. Observations:      1000      AIC:                     3427.
Df Residuals:          995       BIC:                     3452.
Df Model:               5
Covariance Type:       nonrobust
=====
```

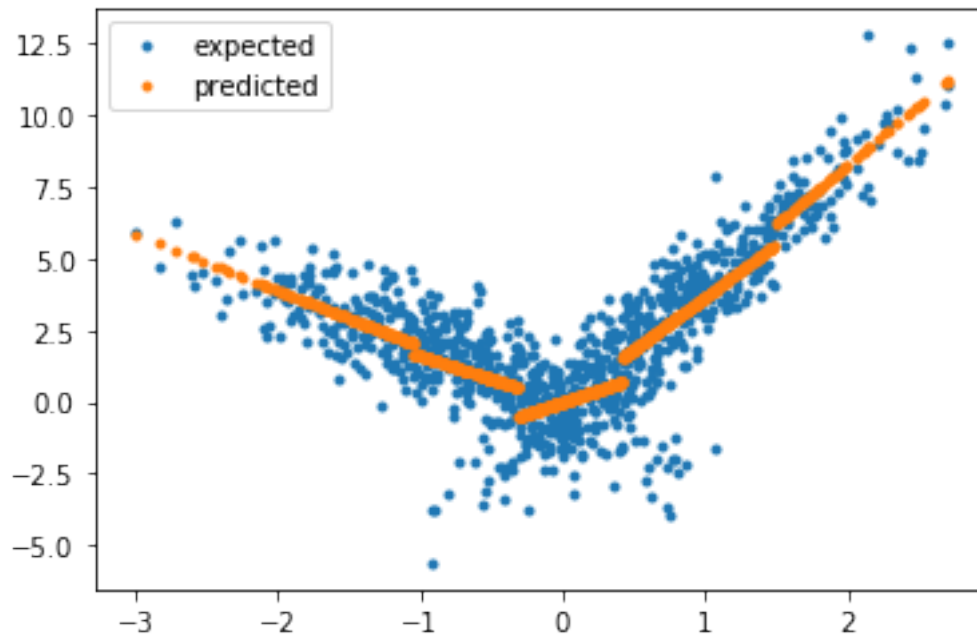
	coef	std err	t	P> t	[0.025	0.975]
x1	-1.9504	0.066	-29.574	0.000	-2.080	-1.821
x2	0.3384	0.148	2.287	0.022	0.048	0.629
x3	3.2628	0.397	8.209	0.000	2.483	4.043
x4	2.0247	0.385	5.260	0.000	1.269	2.780
x5	0.4635	0.119	3.901	0.000	0.230	0.697
=====						
Omnibus:		248.807	Durbin-Watson:		1.984	
Prob(Omnibus):		0.000	Jarque-Bera (JB):		829.417	
Skew:		-1.190	Prob(JB):		7.84e-181	
Kurtosis:		6.774	Cond. No.		20.1	
=====						

Warnings:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
 ""

```
[40]: yp = results.predict(W2)
fig, ax = plt.subplots(1, 1)
ax.plot(X[:, 0], Y, ".", label="expected")
ax.plot(X[:, 0], yp, ".", label="predicted")
ax.legend()
ax.set_title("Régression linéaire par morceaux\nsur un nuage linéaire par morceaux\n" +
            "réduction du nombre de segments\nR2=%f" % results.rsquared);
```

Régression linéaire par morceaux
 sur un nuage linéaire par morceaux
 réduction du nombre de segments
 R2=0.846099



Le coefficient R^2 est quasiment identique pour un nombre de segments moindre. Je me suis amusé à rendre ce code plus générique pour comparer la première étape, le découpage en morceaux, via deux modèles, un arbre de décision et le nouvel objet `KBinsDiscretizer` qui segmente une variable sans tenir compte de la cible. La régression n'est plus nécessaire linéaire : `Piecewise linear regression`. Je me suis également amusé à faire de même pour une classification par morceaux `PiecewiseClassifier`. Celle-ci pose quelques soucis pratiques car toutes les classes ne sont pas forcément représentées dans chaque compartiment...

[41] :